2016 Calculus Huddle — Level 6

1. If $f(x) = |x|$ then $f'(0) =$

   a. 1 b. -1 c. 0

   d. ∞ e. none of these

2. If $y = x\,cos(x)$ then $\frac{dy}{dx} =$

   a. $-sin(x)$ b. $xsin(x) - cos(x)$

   c. $cos(x) - xsin(x)$ d. $cos(x)$

   e. none of these

3. If $f(x) = \frac{x-3}{x+5}$ then the domain of $f^{-1}(x)$ includes all real numbers ***except:***

   a. $x = -5$ b. $x = 1$ c. $x = 3$

   d. $x = \frac{-3}{5}$ e. none of these

4. For the function $y = x^3 + 2x^2 - 15x - 7$ for $-5 \leq x \leq 5$
   the point $(-3, 29)$ is:

   a. a relative maximum b. a relative minimum

   c. an absolute maximum d. an absolute minimum e. none of these

5. For the function $y = x^3 + 2x^2 - 15x - 7$ for $-5 \leq x \leq 5$
   the point $(\frac{5}{3}, \frac{-589}{27})$ is:

   a. a relative maximum b. a relative minimum

   c. an absolute maximum d. an absolute minimum e. none of these

6. $\int_{-5}^{5} \sqrt{25 - x^2}\, dx =$

a. 25π b. 0 c. 12.5π

d. 50π e. none of these

7. The total area bounded by the $x - axis$ and the curve $y = 3x^2 - 3$ between $x = 0$ and $x = 4$ is:

a. 52 b. 66 c. 64

d. 50 e. none of these

8. If $g(x) = \int_0^x (t^3 - 5t)\, dt$ then $g'(x) =$

a. $3x^2 - 5$ b. $t^3 - 5$ c. $3t^2 - 5$

d. $x^3 - 5x$ e. none of these

9. If $y = \frac{2x-7}{5-3x}$ then $y' =$

a. $\frac{31-12x}{(3x-5)^2}$ b. $\frac{-2}{3}$ c. $\frac{-11}{(5-3x)^2}$

d. $\frac{-31}{(5-3x)^2}$ e. none of these

10. The height of a rocket is predicted by the formula $h(t) = 500 + 700t - 16t^2$ where $h(t)$ represents the height, in feet, of the rocket after t seconds. So the instantaneous velocity at $t = 25$ seconds is:

a. 100 miles per hour b. 100 feet per second

c. $-$ 800 feet per second d. $-$ 100 feet per second e. none of these

11. $\int_{-3}^{2}(5x^2-3x+7)dx =$

a. $\frac{605}{6}$ b. $\frac{491}{6}$ c. $\frac{-349}{6}$

d. $\frac{-605}{6}$ e. none of these

12. $\lim_{x\to 3}\frac{x^3-27}{x^2-9} =$

a. 4.5 b. 0 c. 2.5

d. 4 e. none of these

13. Over what open interval(s) is the function $g(x) = x^5 - 15x^3 + 8$ decreasing?

a. $(-3,\infty)$ b. $(-3,0)\cup(3,\infty)$

c. $(-\infty,-3)\cup(3,\infty)$ d. $(-3,3)$

e. none of these

14. $\int_{-4}^{2}|x|\,dx =$

a. 10 b. -6 c. 6

d. -10 e. none of these

15. A cylinder has a radius of five feet and a height of 12 feet. Water is going into the cylinder at a rate of 0.5 cubic feet per minute. How fast is the depth of the water changing?

a. 50π feet per minute b. $\frac{1}{12.5\pi}$ feet per minute

c. $\frac{1}{50\pi}$ feet per second d. 12.5π feet per second

e. none of these

16. $\lim_{h \to 0} \frac{tan(\frac{\pi}{4}+h)-1}{h} =$

a. 2 b. 1 c. $\sqrt{2}$

d. ∞ e. none of these

17. $\int_e^{e^3} \frac{dx}{ln(x^x)} =$

a. e^3 b. $ln(3)$ c. $e^3 - e$

d. e^2 e. none of these

18. If $y = 5^{x^2}$ then $\frac{dy}{dx} =$

a. $5^{x^2} ln(5)$ b. $2x5^{x^2}$ c. $ln(25)x5^{x^2}$

d. $5^{x^2} ln(x^2)$ e. none of these

19. The absolute maximum value of the function $f(x) = \frac{3x}{x^2+4}$ is:

a. 0.75 b. 2 c. 3

d. There is no absolute maximum. e. none of these

20. The vertical asymptote(s) of $y = \frac{x^2+5x}{x^2-25}$ is (are):

a. $x = 5$ and $x = -5$ b. $y = 1$ c. $x = 5$

d. $x = -5$ e. none of these

21. $\lim_{n\to\infty}(1+\frac{3}{n})^{5n} =$

a. e^5 b. e^3 c. e^{15}

d. 0 e. none of these

22. The slope of the line tangent to $y = x^x$ at $x = 2$ is:

a. undefined b. $4 + ln(2)$ c. $4ln(2)$

d. $4 + ln(16)$ e. none of these

23. For the function $g(x) = x + sin(2x)$ on the interval $0 \leq x \leq 2\pi$, the number of horizontal tangents is:

a. 4 b. 2 c. 1

d. 0 e. none of these

24. Let $f(x) = \begin{cases} x^2 - 3x & \text{if } x \leq -2 \\ \frac{x-2a^2}{x+5} & \text{if } x > -2 \end{cases}$

For what real value(s) of a will $f(x)$ be continuous at $x = -2$?

a. $a = \pm 4$ b. $a = 4$ c. $a = 16$

d. $a = -4$ e. none of these

25. If $y = \frac{sin(x)}{1+cos(x)}$ then $\frac{dy}{dx} =$

a. $\frac{cos(x)}{(1+cos(x))^2}$ b. $cos(x) + 1$ c. $\frac{1+cos(x)}{sin^2(x)}$

d. $csc^2(x) - csc(x)cot(x)$ e. none of these

26. How many inflection points does the graph of $y = e^{x^3}$ have?

a. one b. two c. three

d. none e. none of these

27. A tower with a light on the top is 400 feet above, and perpendicular to, the water level. A boat is slowly moving away from the base of the tower at a constant rate of 20 feet per minute. The light is shining on the water level next to the boat. How fast is the angle between the beam of light and the tower changing when the boat is 300 feet from the base of the tower?

a. 0.032 radians per second b. 0.018 radians per minute

c. $\frac{3.24}{\pi}$ degrees per minute d. $\frac{5.76}{\pi}$ degrees per minute

e. none of these

28. $\int_0^1 \frac{dx}{\sqrt[3]{5x+3}} =$

a. $6 - 1.5\sqrt[3]{9}$ b. $1.2 - 0.3\sqrt{27}$ c. $1.2 - 0.3\sqrt[3]{9}$

d. $0.8 - 0.2\sqrt[3]{9}$ e. none of these

29. $\int_{ln(2)}^{ln(5)} (e^x + e^{-x})dx =$

a. 2.7 b. 3.3 c. 2.3

d. 3 e. none of these

30. For the relation defined by $x^3y^2 + 2x - 5y = 10$, $y' =$

a. $\frac{1-3y}{2x^3y}$

b. $\frac{-3x^2y^2+3}{2x^3y}$

c. $\frac{3x^2y^2+2}{5-2x^3y}$

d. $\frac{-3x^2-2}{2y-5}$

e. none of these